United States Department of Agriculture
AGRICULTURAL MARKETING SERVICE

THEORETICAL ASPECTS
OF THE
USE OF THE CROP METER

NUMBER II
OF A SERIES OF
ANALYSES OF SAMPLE FARM DATA

Prepared with the assistance of the Work Projects Administration
for the City of New York

O. P. No. 765-97-3-16

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The assistance given by the Board of Education of the City of New York, in the publication of this report is gratefully acknowledged.

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EXPLANATORY NOTE

This report gives the results of an investigation to determine the validity of assumptions made in translating linear measurements of crop frontages on highways into estimates of corresponding crop acreages in the region traversed. Such a study must obviously be based upon a universe of known constitution so that various phases of the problem can be examined in detail. Agricultural data obtained from usual sources are not sufficiently extensive for an analysis of this kind but aerial survey photographs made available by the Agricultural Adjustment Administration provided a good source of experimental material for the purpose at hand. After the crops in the various individual fields shown on the photographs were identified by visits to the farm operators concerned and the highways traversing the region were traced on the photographs, an ideal universe for study was made available.

This study was undertaken in November 1938 by the Bureau of Agricultural Economics with the assistance of the Works Progress Administration of New York City (now Work Projects Administration) as Official Project No. 765-97-3-16 and was completed in June 1939. With the establishment of the Agricultural Marketing Service on July 1, 1939, the work was transferred to that agency of the Department of Agriculture. The study was made under the general supervision of C. F. Sarle, Principal Economist, and A. J. King, Agricultural Statistician, both of the Agricultural Marketing Service. Glenn D. Simpson, Associate Statistician, representing the Service on several phases of agricultural research, was largely responsible for the administrative details of the project. C. B. Lawrence, J., Coordinator of Statistical Projects of the Work Projects Administration, furnished many helpful suggestions and criticisms.

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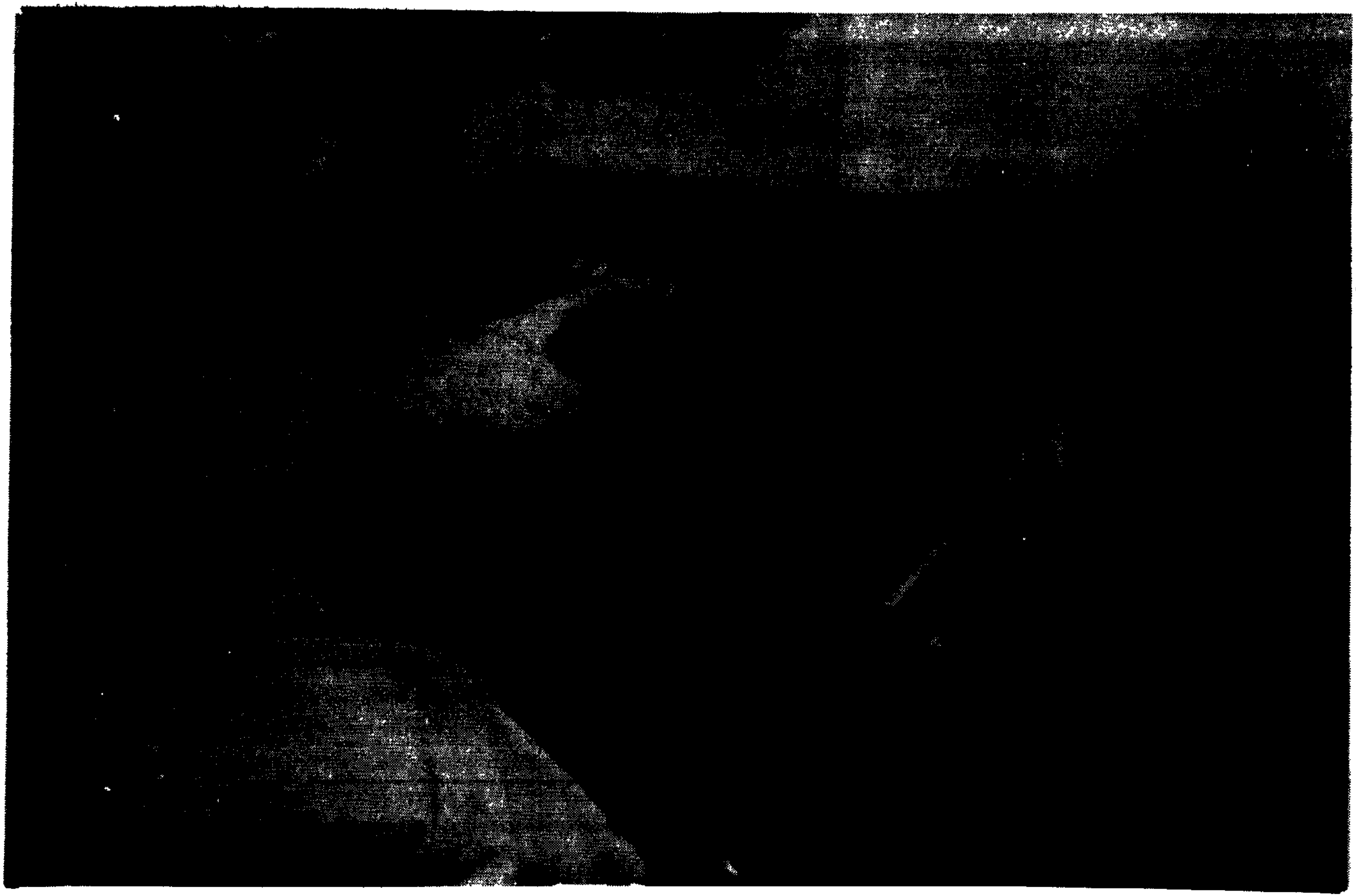

Figure 1.—Crop meter in operation. Small dials register frontages of different crops and large dial in lower right-hand corner registers total distance driven.

THEORETICAL ASPECTS OF THE USE OF THE CROP METER

By Walter A. Hendricks, Agricultural Statistician

INTRODUCTION

Most agricultural statisticians who have utilized sample data from crop reporters as a basis for estimates of crop acreages have appreciated the need for a more objective method of obtaining data. A number of years ago, the field agent for South Carolina of the Bureau of Crop Estimates (now known as the Agricultural Statistics Division, Agricultural Marketing Service) made counts of the number of individual fields throughout the State in which various crops were growing. The method was crude but showed indications of promise. Field counts from trains were used in a number of States for several years. Since the average size of the fields is as important as their number, the method was extended to include an estimate of the total frontage of a given crop along the railroad right of way. This was accomplished by recording the number of telephone or telegraph poles opposite the fields planted to each crop, and was known as the "pole count."

These rather crude procedures eventually led to a more refined method of measuring changes in crop acreages that was developed by the agricultural statistician for Mississippi about 15 years ago, and has become increasingly popular as an objective method of supplementing other sources of information available to the Department of Agriculture. This method is based on the measurement, in linear units, of the frontage of cotton, corn, wheat, and other crops along a highway, the unit being taken, for convenience, as 0.02 of a mile. The measurement is performed with a "crop meter" attached to the instrument panel of an automobile and driven by a speedometer cable, as shown in figure 1.

The operation of the instrument requires very little explanation. A large dial registers the total distance driven and a number of smaller dials register the frontage measurements of various crops. The appropriate dial is put in gear by means of a push button, when the automobile is flush with the first corner of the field, and continues to register until a release button is pushed, when the frontage is completely measured. In the course of the trip, the total frontage for each of several crops accumulates in the various dials and one can compute the ratio of the frontage of each crop to the total length of the route.

It has been found that the total acreage of a crop, in the region traversed, tends to be proportional to the ratio obtained by dividing the total frontage measured on highways, by the length of the route covered. This relationship is illustrated in figure 2, where the South Carolina cotton acreages for the years 1928-39, inclusive, have been plotted against the crop-meter ratios for the same years. In actual practice, an estimate of the acreage of a crop based on crop-meter readings is often obtained by computing the percent increase in relative frontage above the amount obtained on the same route the preceding year and equating this ratio to the percent increase in total acreage. This procedure eliminates the necessity for determining the regression of total acreage on relative frontage and eliminates some sources of variability from the data. These possible sources of variability are discussed later in the report. From a few hundred to 7,500 miles of route are covered each year in the States where the crop-meter is used and an attempt is made to keep the routes as nearly identical from year to year as possible.

The proportionality between total acreage in a region and relative frontage on highways is an interesting relationship when one considers the fact that one variable is expressed in linear units while the other is expressed in units of area. One of the first attempts to explain this relationship may be found in some notes, prepared by S. A. Jones and J. B. Shepard of the Agricultural Marketing Service, that were made available to statisticians in the crop and livestock reporting service by means of a field memorandum, issued under date of February 12, 1927. The mathematical part of the discussion is an attempt to show that the relative frontage of a given crop on highways tends to be equal to the relative area occupied by that crop in the tract of land traversed.

FIGURE 2

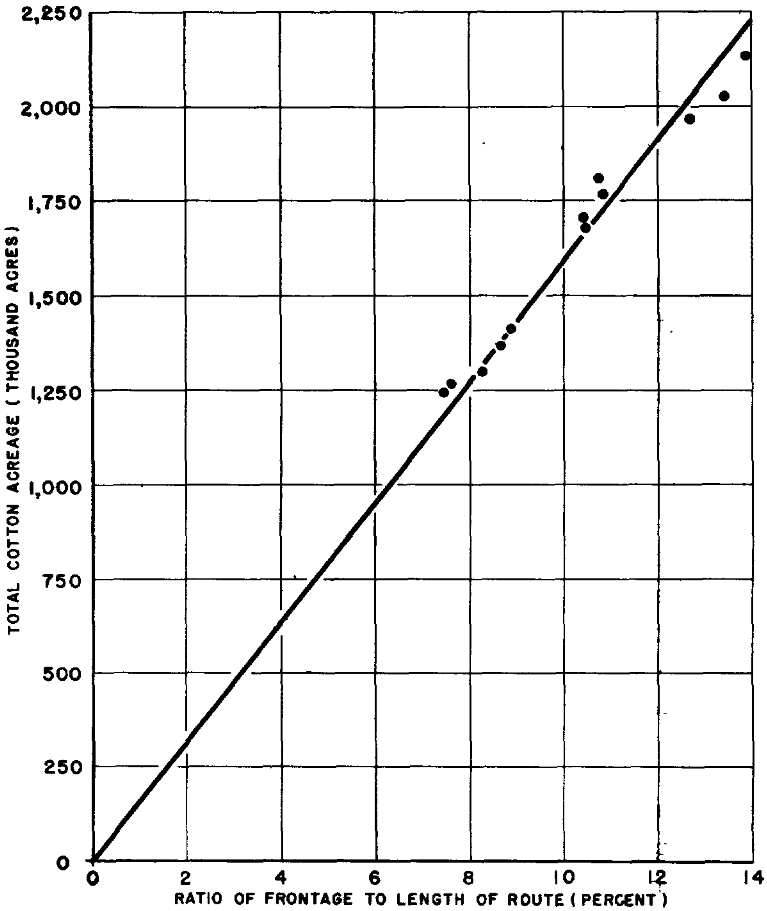

Figure 2.—Relation between South Carolina planted cotton acreage 1928-39 and the ratio of cotton frontage to length of route. The position of each dot represents the frontage ratio on the crop-meter route as compared with the total cotton acreage in the State for a given year. The straight line through the origin was fitted so that the sum of the vertical deviations of the 12 dots from the line is equal to zero.

This conclusion was based on two fundamental assumptions. The first assumption is that, on the average, the areas of individual fields tend to be equal to the squares of their frontages when areas and frontages are expressed in comparable units. This assumption appears to be reasonable provided there is no consistent tendency for either a long side, or a short side of a field to be laid off along the side of the highway. The second assumption is that the probability that a field of a given size will lie along one edge of a square tract of land and be measured, is equal to the square root of the relative area occupied by such a field in the square tract of land under consideration. The implications of this assumption can be visualized by referring to figure 3.

Figure 3.—Relation between the number of square units of area in a square mile of land and the number of such units fronting on a mile of road.

FIGURE 3

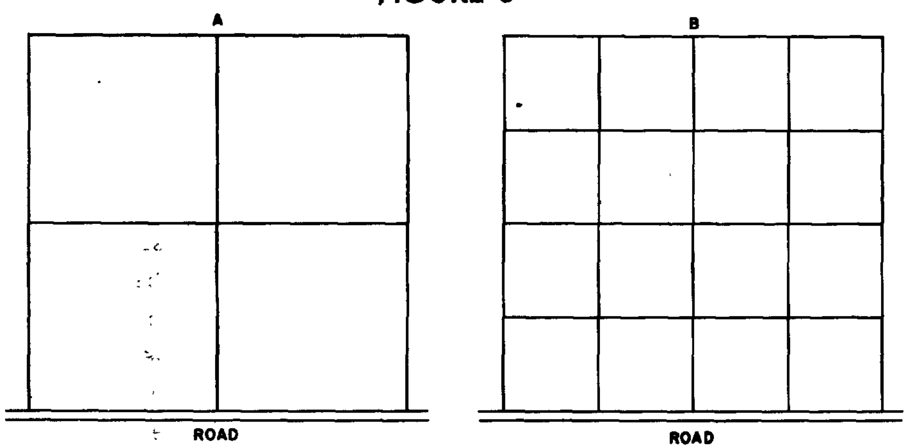

Figure 3 represents two tracts of land, each a mile square, fronting on a highway. Tract A is divided into 4 equal square units and tract B is divided into 16 equal square units. Tract A has 2 units fronting on the road while tract B has 4 units fronting on the road. If a field 1/4 of a square mile in area can assume only one of the 4 positions shown in figure 3-A, the probability that the field will front on the road is obviously

2/4 or 1/2 which is equal to $\sqrt{1/4}$. If a field 1/16 of a square mile in area can assume only one of the 16 positions shown in figure 3-B, the probability that the field will front on the road is 4/16 or 1/4 which is equal to $\sqrt{1/16}$.

It is evident that, if the validity of the second assumption given above is to be conceded, one must be willing to postulate a restriction on the number of possible positions that a field can occupy in a given tract of land. Such a restriction might be effected by the presence of other crops. For instance, if in figure 3-A one assumes that all four quarter-sections are planted to crops, such a restriction must certainly be in operation. If one plot is a wheat field, one a corn field, one a rye field, and the fourth a hay field, there is obviously only one chance in two that the wheat field will front on the road. If the entire tract is not used, however, such a restriction will not take place. This being the case, it appeared desirable to learn the extent to which the simple hypothesis used to explain relationships, such as the one shown in figure 2, is justified in fact.

Nature of the Present Study

In testing the validity of any hypothesis, one is usually concerned with deducing the necessary consequences of that hypothesis, and noting the extent to which observed phenomena are in agreement with what is expected under the hypothesis. It may be stated at the beginning that the results to be presented in this publication confirm the essential features of the assumptions under consideration. The necessary data were obtained from aerial survey photographs, made in 1937, for the following counties on which crop identification had been made:

Washington County, Indiana
Wayne County, Indiana
Jefferson County, Iowa
McLeod County, Minnesota
Gentry County, Missouri
Harlan County, Nebraska
Dane County, Wisconsin
Rock County, Wisconsin

The study was made on a county basis and the work was limited to a consideration of only three crops—corn, wheat, and alfalfa. Corn and wheat are major crops in practically all the eight counties, while alfalfa is a comparatively minor crop. Several highways through each county were identified on road maps and traced on the photographs. An attempt was made to obtain a number of approximately parallel routes going north and south in each county, and a number of approximately parallel routes going east and west in each county. This was accomplished fairly successfully in all counties except Dane County, where highways tend to radiate from Madison, the centrally located State capital of Wisconsin. However, a sufficiently large number of routes was available so that Dane County was covered as thoroughly as the others.

The various phases of the study herein reported deal with relationships that one would expect to find if the fundamental hypothesis under test is valid. Five such relationships were tested and may be listed briefly as follows:

1. The relation between frontages and areas of individual fields.
2. The relation between total crop acreages in each county and the corresponding acreages computed from frontage measurements.
3. The relation between the observed number of fields per mile of route and the number expected under the hypothesis.
4. The relation between the average size of fields in the county and the average size of those fields fronting on the routes.
5. The relation between the observed variability of relative frontages and the variability expected under the hypothesis.

These relationships are discussed in detail in the succeeding sections of this report.

Relation Between Frontages and Areas of Individual Fields

The first assumption to be tested was the one stating that, on the average, the areas of individual fields tend to be equal to the squares of their frontages. From the point of view of the subsequent discussion it is more convenient to restate this assumption by saying that frontages of individual fields tend to be equal to the square roots of their areas.

If there are $n_1, n_2, —, n_p$ fields with areas $A_1, A_2, —, A_p$ and frontages $F_1, F_2, —, F_p$, respectively, where areas are expressed in square miles and frontages in miles, a proportionality between the frontage of an individual field and the square root of its area can be represented by the equation,

$$F_i = c_1 \sqrt{A_i} \qquad (1)$$

The hypothesis under test states that c_1 is equal to unity.

To test the relation indicated by equation (1), a route was chosen at random in each county and the area and frontage of each of 100 successive fields on this route, growing either corn, wheat, or alfalfa, were measured. It was found that, for each county, the areas of individual fields tended to be proportional to the frontages rather than to the squares of the frontages. In other words, it appeared that on any given segment of road there is a tendency for depths and frontages of individual fields to vary independently about their average values. The factor of proportionality, however, was found to depend on the average size of the fields for each county so that the average areas were proportional to the squares of the average frontages. The results of this phase of the study are summarized in table 1 and presented graphically in figure 4.

It is evident that the data in the last column of table 1 are not correlated with the average areas of the fields. The average of these values, 201.173, provides an estimate of c_1 in equation (1). If areas and frontages are expressed in comparable units, this constant has the value 0.96389. Its standard error is 0.01331. Thus, the value of c_1 appears

FIGURE 4

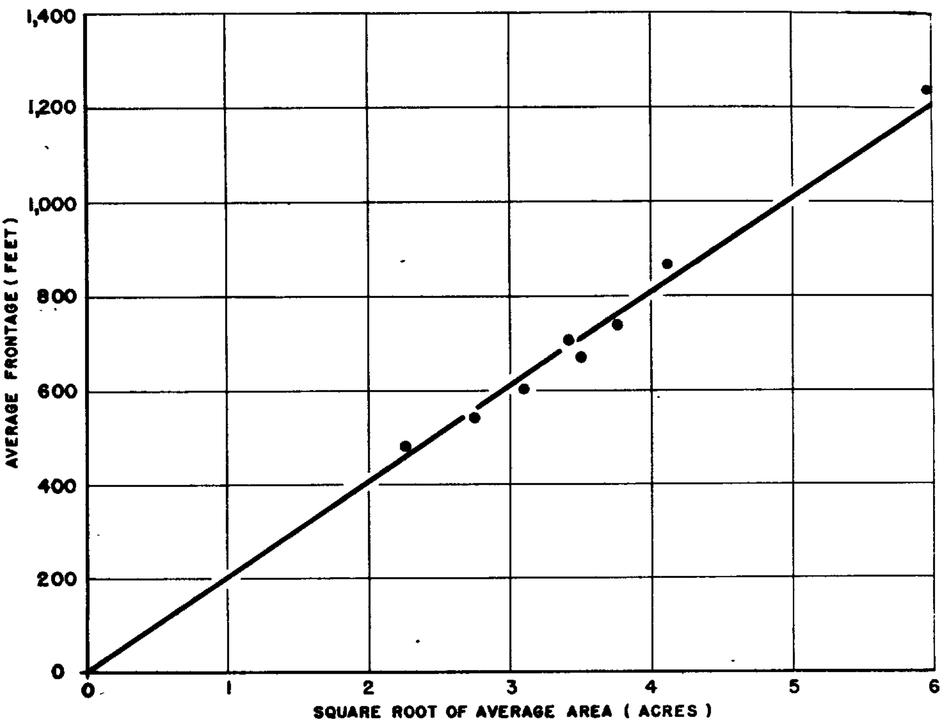

Figure 4.—Relation between average frontage and average area of individual fields on a route. The position of each dot represents the square root of the average area of 100 individual fields on a random route in a county as compared with the average frontage of those fields. Each dot represents data for a different county. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

Table 1.—Relation Between Area and Frontage for 100 Individual Fields on a Random Route Through Each County

COUNTY	Av. area	Av. frontage	Av. frontage / Av. area	Av. frontage / Square root of Av. area
	Acres	Feet		
Dane, Wis.	5.127	476.30	92.900	210.35
McLeod, Minn.	7.559	539.59	71.384	196.26
Wash., Ind.	9.579	602.10	62.856	194.54
Gentry, Mo.	11.699	701.23	59.939	205.01
Rock, Wis.	12.244	667.11	54.485	190.65
Wayne, Ind.	14.172	734.70	51.842	195.17
Jefferson, Iowa	16.959	865.70	51.047	210.22
Harlan, Neb.	35.748	1238.75	34.652	207.18
Average				201.173

to be significantly different from unity. However, in the absence of any logical explanation of such a difference, little importance can be attached to it. The difference might have arisen as a sampling fluctuation, though it is somewhat larger than one might expect. The observed value was used in all the subsequent computations involving c_1 to avoid the possibility of introducing an error.

Observed Crop Acreage and Acreages Computed from Frontage Measurements

The fundamental data required for studying the relation between the crop acreages in a county and the corresponding frontage measurements on highways are given in table 2.

The geographic areas of the counties given in table 2 are somewhat empirical. In some cases the county boundaries could not be clearly distinguished on the aerial photographs, and in some cases small portions of a county had to be neglected because crop identification was incomplete. In all cases, however, the areas given are the measured areas to which the field counts and crop measurements apply. The data in the

Table 2.—Data Relating to Field Counts and Crop Measurements for Each County

County	Geographic area of county*	Total Length of all routes in county	Crop	No. fields in county	Area of all fields in county	No. fields on all routes in county	Area of fields on all routes in county	Crop frontage on all routes in county
	Sq. miles	miles		number	acres	number	acres	feet
Dane, Wis.	1219.49	299.53	Corn	16,853	138,415	867	7,580.7	492,336
			Wheat	674	2,799	17	59.0	6,864
			Alfalfa	8,694	48,354	489	3,005.2	255,578
McLeod, Minn.	500.77	149.53	Corn	5,003	57,832	308	4,120.8	215,584
			Wheat	2,327	20,837	117	1,092.3	72,239
			Alfalfa	3,507	15,568	238	1,217.7	130,383
Washington, Ind.	500.12	183.60	Corn	5,293	36,609	397	3,538.4	247,835
			Wheat	1,736	17,266	153	1,689.7	95,475
			Alfalfa	471	2,986	65	412.4	39,420
Gentry, Mo.	489.35	217.98	Corn	4,491	45,047	338	3,859.5	229,213
			Wheat	2,308	29,005	190	2,395.7	134,951
			Alfalfa	504	2,968	37	236.9	22,144
Rock, Wis.	716.94	224.13	Corn	9,264	107,439	685	8,346.5	431,787
			Wheat	684	4,121	46	259.1	23,760
			Alfalfa	1,407	9,120	116	705.5	55,930
Wayne, Ind.	374.59	149.72	Corn	4,776	63,636	401	5,655.7	305,479
			Wheat	2,268	36,825	226	3,466.7	178,926
			Alfalfa	1,388	10,740	134	1,051.0	81,613
Jefferson, Iowa	435.43	173.52	Corn	4,489	62,149	492	8,101.0	386,095
			Wheat	864	9,512	89	1,065.7	62,520
			Alfalfa	191	1,239	17	105.9	8,350
Harlan, Nebr.	572.19	158.047	Corn	2,732	71,678	311	9,282.8	304,354
			Wheat	2,216	83,222	341	15,050.6	421,107
			Alfalfa	395	3,375	35	270.9	19,672

*Entire area included within county boundaries last three columns of the table refer to counts and measurements made on both sides of the routes. The numbers of fields, areas, and frontages appeared to be approximately the same on each side of every route. Thus, any formula requiring counts or measurements on only one side of a route can be applied if the data in the last three columns of the table are divided by 2.

The nature of the mathematical relationship between crop acreages and frontage measurements may now be discussed from the point of view of the hypothesis under test. If in a square mile of land, there are n_1, n_2, —, n_p fields with areas A_1, A_2, —, A_p, where areas are expressed in square miles; and the probability that a field of area A_i will lie on one side of a mile of route is proportional to $\sqrt{A_i}$, the number of fields of that size on one side of the route may be represented by the equation,

$$n_1' = c_2 \, n_i \sqrt{A_i} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where n_i' is the number of fields of a given size on one side of a mile of route and n_i is the number of such fields per square mile of area. The total area of all fields in a square mile is equal to the sum of the areas of the individual fields, or $S(n_i \, A_i)$. The total frontage of those fields that fall on one side of a mile of route is $S(n_i \, F_i)$. By making use of equations (1) and (2), this expression can be written in the form $S(c_2 \, n_i \sqrt{A_i} \; c_1 \sqrt{A_i})$ or $c_1 \, c_2 \, S(n_i \; A_i)$. The ratio of the total area of the fields in the square mile to the total frontage of those that fall on one side of a mile of route is, therefore, equal to $1/(c_1 c_2)$.

It is important to notice that equation (1) is concerned with the relationship between frontage and area of fields which actually lie on the route. Equation (2) gives the relation between the number of fields in a square mile of area and the number that lie on a mile of route. Combining these two equations establishes a relationship between the frontage of the fields which actually lie on a mile of route and the area of all fields in a square mile, all of which will not be on the route.

As a preliminary step in the analysis of the data from this point of view, the relative frontage of each of the three crops was computed for each county by dividing the total frontage by the length of the route. The relative frontage was multiplied by the area of the county to obtain an estimate of the crop acreage in the county under the assumption of equality of relative frontage and relative acreage. The results are summarized in table 3.

Table 3.—Observed Crop Acreages and Acreages Computed by Assuming Equality Between Relative Frontages and Relative Acreages

COUNTY	CORN		WHEAT		ALFALFA	
	Observed	Computed	Observed	Computed	Observed	Computed
	acres	acres	acres	acres	acres	acres
Dane, Wis.	138,415	121,483	2,799	1,694	48,354	63,063
McLeod, Minn.	57,832	43,756	20,837	14,662	15,568	26,463
Washington, Ind.	36,609	40,915	17,266	15,762	2,986	6,508
Gentry, Mo.	45,047	31,186	29,005	18,361	2,968	3,013
Rock, Wis.	107,439	83,708	4,121	4,606	9,120	10,843
Wayne, Ind.	63,636	46,321	36,825	27,131	10,740	12,375
Jeff., Iowa	62,149	58,719	9,512	9,508	1,239	1,270
Harlan, Neb.	71,678	66,781	83,222	92,398	3,375	4,316
Total	582,805	492,869	203,587	184,122	94,350	127,851

The total observed acreage in the three crops is 880,742, while the computed acreage is only 804,842. This indicates that the value of c_2 in equation (2) is not equal to unity. Furthermore, it is apparent that the corn and wheat acreages were consistently underestimated whereas the alfalfa acreages were consistently overestimated. This indicates that the value of c_2 tends to vary from crop to crop.

It was conceivable that the value of c might vary from county to county as well as from crop to crop. The crop-to-crop variation can be explained by a tendency for certain crops to be planted near highways. A county-to-county variation could be explained by differences in the topography of the counties, or differences in the nature of the highways that might easily affect the probability of encountering crops in general. Some highways tend to pass through nonagricultural regions; others tend to limit themselves to agricultural regions.

In order to obtain some information on these points, the ratio of the observed acreage to the corresponding computed acreage for each crop was obtained for each of the 8 counties. The resulting 24 ratios were investigated by analysis of variance. One need reflect only a moment to conclude that, if there is a county-to-county variation and a crop-to-crop variation, the composite effect of these sources of variability on the ratio for a given crop in a given county is multiplicative rather than additive. For this reason, the analysis of variance was performed with the logarithms of the ratios rather than with the ratios themselves. The results of the analysis are summarized in table 4.

Table 4.—Analysis of Variance of Log $\frac{\text{Relative acreage}}{\text{Relative frontage}}$

Source of variability	Degrees of freedom	Sum of squares	Mean square	F
Between crops	2	0.20409	0.102045	14.354
Between counties	7	.10650	.015214	2.140
Error	14	.09953	.007109	
Total	23	0.41012	0.017831	

The variance between crops is highly significant but the variance between counties is not significant. These results indicated that the tendency for certain crops to be planted on highways was fairly consistent from county to county. The county-to-county variation was not sufficiently large for all crops, however, to make it worth while to attempt an adjustment on the computed acreages in table 3 that took this factor into consideration. An adjustment for the crop bias is all that appeared to be required at this stage of the analysis.

The ratio of total observed acreage to total computed acreage was obtained for each crop. These ratios have the values

Corn 1.18247
Wheat 1.10572
Alfalfa 0.73797

Figures 5, 6, and 7 show the effects of this crop bias in graphical form. When the data in table 3 were adjusted for the bias, the results shown in table 5 were obtained.

Table 5.—Observed Crop Acreages and Computed Acreages Adjusted fro Crop Bias

County	Corn		Wheat		Alfalfa	
	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres
Dane, Wis.	138,415	143,650	2,799	1,873	48,354	46,539
McLeod, Minn.	57,832	51,740	20,837	16,212	15,568	19,529
Washington, Ind.	36,609	48,381	17,266	17,428	2,986	4,803
Gentry, Mo.	45,047	36,877	29,005	20,302	2,968	2,224
Rock, Wis.	107,439	98,982	4,121	5,043	9,120	8,002
Wayne, Ind.	63,636	54,773	36,825	29,999	10,740	9,132
Jefferson, Iowa	62,149	69,433	9,512	10,513	1,239	937
Harlan, Nebraska	71,678	78,967	83,222	102,166	3,375	3,185
Total	582,805	582,803	203,587	203,586	94,350	94,351

FIGURE 5

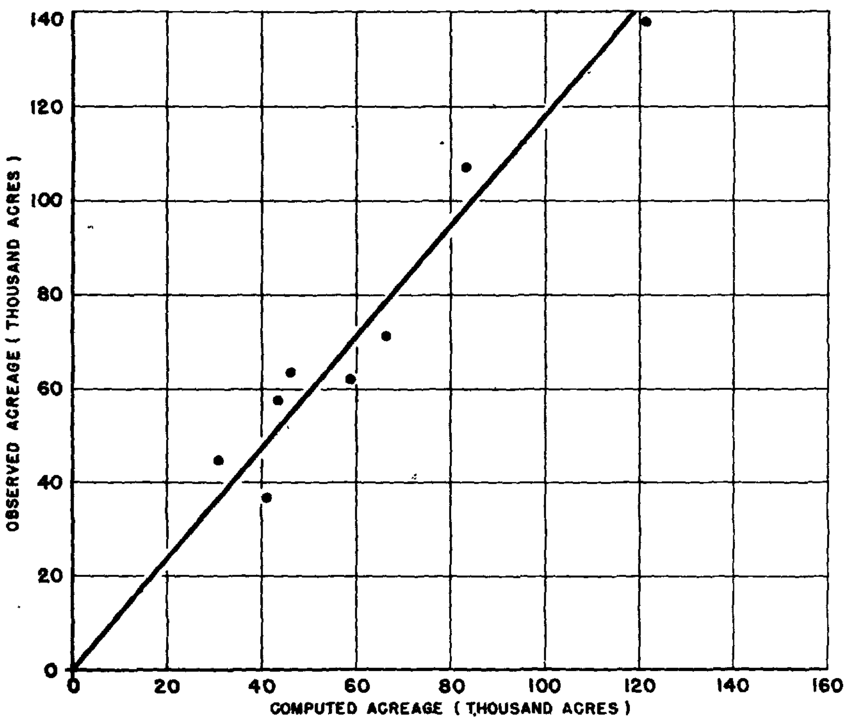

Figure 5.—Total corn acreage in each county and acreage computed b assuming equality between relative acreage and relative frontage. Th position of each dot represents the computed acreage in a county compared with the observed acreage. The straight line through th origin was fitted so that the sum of the vertical deviations of the 8 do from the line is equal to zero.

FIGURE 6

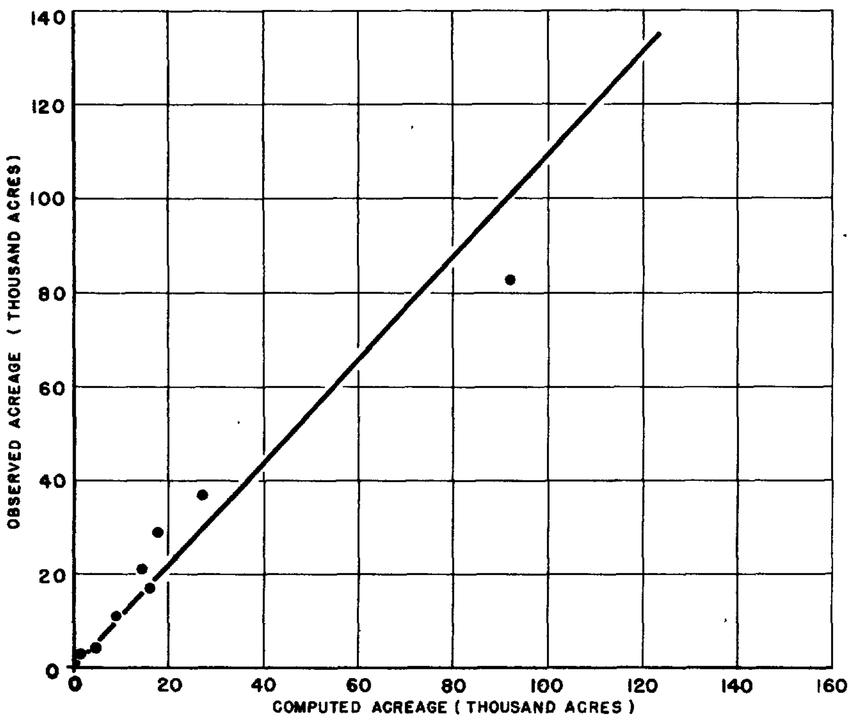

Figure 6.—Total wheat acreage in each county and acreage computed by assuming equality between relative acreage and relative frontage. The position of each dot represents the computed acreage in a county as compared with the observed acreage. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

FIGURE 7

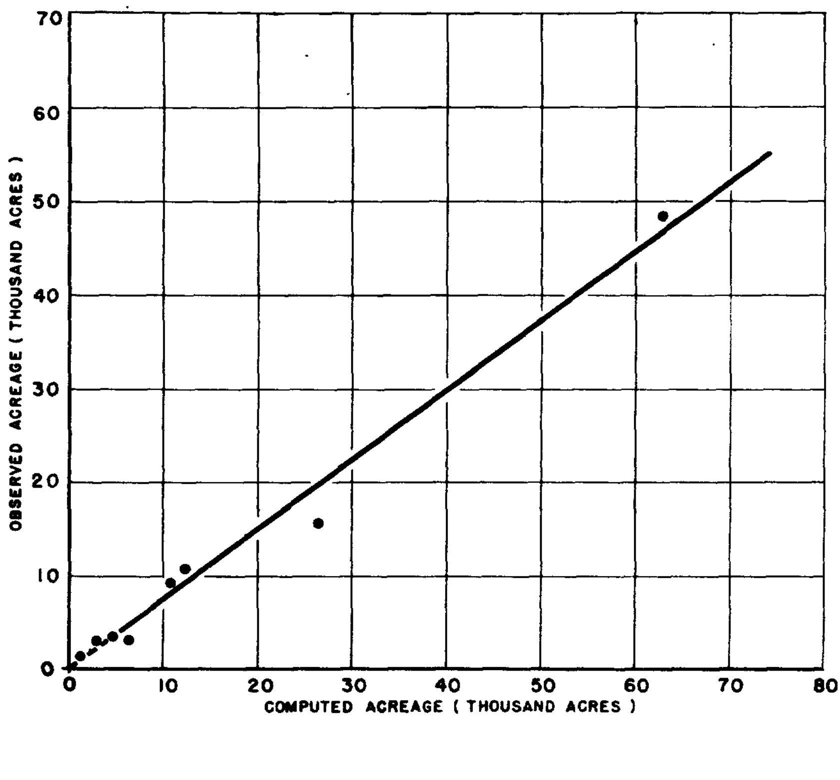

Figure 7.—Total alfalfa acreage in each county and acreage computed by assuming equality between relative acreage and relative frontage. The position of each dot represents the computed acreage in a county as compared with the observed acreage. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

Number and Average Size of Fields per Mile of Route in Relation to Distribution of Field Sizes in County

The results presented so far show that the relative acreage of a given crop in a county is proportional to the relative frontage of that crop on highways. The analysis would not be complete without a study of the relationship between the number of fields per square mile of area in a county and the number of fields per mile of route in that county for each crop, together with a consideration of the average size of the fields on the routes as compared with the average for the entire county. According to equation (2), the number of fields of a given size which can be expected on a mile of route is equal to the product $c_2 \ n_i \sqrt{A_i}$, in which A_i is the area of a single field, expressed in square miles, and n_i is the number of fields of that size per square mile of area in the county.

The value of c_2 for each crop can be estimated from the three crop bias ratios given above. According to the theory, each of these ratios is the value of the quantity $\frac{1}{c_1 c_2}$ for the corresponding crop. If the value of c_1 is taken to be 0.96389, the values of c_2 computed from the above ratios are

Corn	0.87737
Wheat	0.93827
Alfalfa	1.40584

It would appear to be a simple matter to apply equation (2) to the observed data. If a county contains n fields of a given crop per square mile of area and n_i represents the number of fields of any given area A_i , the number of fields in that crop per mile of route should be equal to $c_2 S (n_i \sqrt{A_i})$. However, as a practical matter, the computation of the value of this expression would be burdensome. The procedure followed in the present study does not differ in principle from that indicated above, but can be applied with much less labor. Briefly, the method consisted of deriving a mathematical equation to represent the frequency distribution of individual fields for each crop in a county and replacing the sum in the expression $c_2 \ S(n_i \sqrt{A_i})$ by a definite integral.

The function that seemed appropriate to represent this frequency distribution was a Pearsonian Type III Curve of the form,

$$df_A = \frac{a^{b+1}}{\Gamma(b+1)} e^{-aA} A^b dA \quad \ldots \ldots \ldots \ldots \ldots \quad (3)$$

in which a and b are parameters whose numerical values in any given instance depend upon the average size of the fields and the coefficient of variation* of the individual fields.

The average size of the fields for each crop in each county was computed from the data in table 2 but the coefficient of variation was computed indirectly. According to the theory, the probability that a field lies on a mile of route is proportional to the square root of its area. If this is true, the proportion of large fields found on the routes in a county will be greater than the proportion in the county as a whole. If the frequency distribution of the fields in the county is given by equation (3) it may be shown that the coefficient of variation is given by the formula,

$$v^2 = \frac{2(\bar{A}_r - \bar{A})}{\bar{A}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (4)$$

in which v represents the coefficient of variation, $\bar{A}_r$ represents the average area of the fields on the routes, and $\bar{A}$ represents the average area of the fields in the county. The derivation of this formula is given in the Appendix to this report for the benefit of those interested in the mathematical aspects of the problem.

It should be noted that the average area of the fields on the routes must necessarily be greater than the average area of all the fields in the county if the fundamental theory underlying the use of the crop meter

* In this report the "coefficient of variation" is defined as the ratio of the standard deviation to the arithmetic mean.

is valid. The author has frequently heard statements to the effect that an efficient use of the crop meter presupposes that the fields on the routes are a "good" or a "fair" or a "representative" sample of the fields in the region. Such statements are inconsistent with the theory under discussion. One cannot, at one stage of an argument, assume that large fields have a greater probability of being encountered on a route than small fields and, at another stage, that the fields on the route are a representative sample of the fields in the region traversed by the route.

An examination of the data indicated that the value of v tended tp be constant from crop to crop and from county to county. Equation (4) shows that, if v is constant, the relative bias in the average area of the fields on the routes is also constant. In view of these considerations, the data for corn, wheat, and alfalfa were combined and the relative bias in the average area of the fields on the routes was computed from the combined data for each county. The results are summarized in table 6 and are presented graphically in figure 8.

Table 6.—Average Area of Corn, Wheat, and Alfalfa Fields in Each County and on all Routes in Each County

County	Average area of fields in county	Average area fields on routes	Bias	
	acres	acres	acres	relative
Dane, Wis.	7.230	7.753	0.523	0.07234
McLeod, Minn.	8.696	9.700	1.004	.11546
Washington, Ind.	7.581	9.172	1.591	.20987
Gentry, Mo.	10.546	11.490	.944	.08951
Rock, Wis.	10.628	10.993	.365	.03434
Wayne, Ind.	13.188	13.368	.180	.01365
Jefferson, Iowa	13.149	15.506	2.357	.17925
Harlan, Nebraska	29.623	35.814	6.191	.20899
Average				0.11543

FIGURE 8

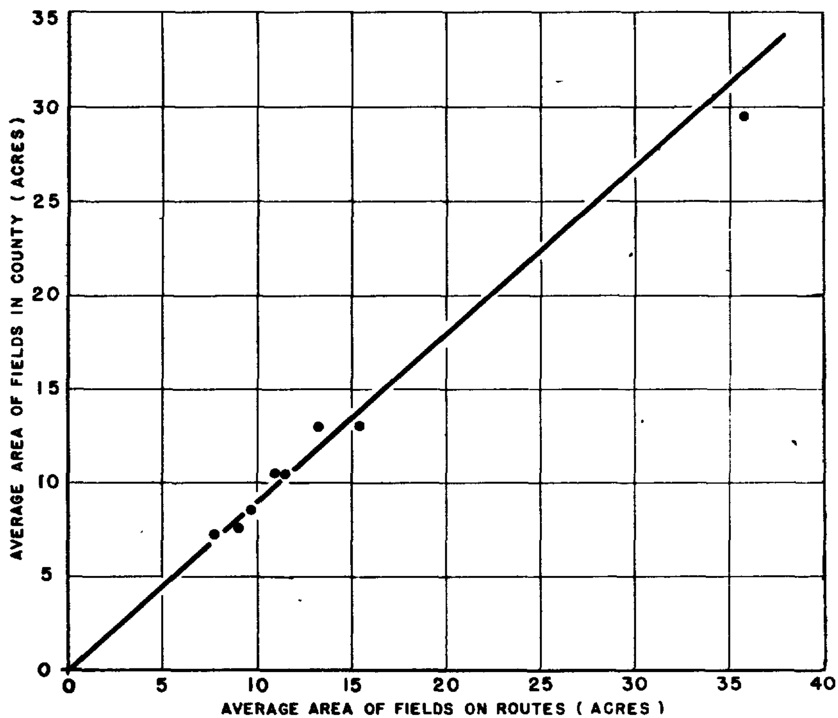

Figure 8.—Average area of fields in each county and average area of fields on routes in each county. The position of each dot represents the average area of the fields on the routes in a county as compared with the average area of all fields in the county. Only fields growing either corn, wheat, or alfalfa were considered. The straight line through the origin was fitted so that the sum of the vertical deviations of the 8 dots from the line is equal to zero.

The average relative bias shown in table 6 is 0.11543. The coefficient of variation of individual fields, therefore, is equal to the square root of 0.23086 or 0.48048. This value of v, together with the average area of the fields for each crop, provides all the information required tc compute the values of a and b for each crop by counties. Thus, the frequency distribution of the size of field is established for each crop in each county. By means of these distributions it may be shown that the number of fields that can be expected on a mile of route is given by the formula,

$$n_r = 0.97163\ c_2\ n\sqrt{\bar{A}} \quad . . . . (5)$$

in which n_r is the number of fields of a given crop per mile of route, n is the number of such fields per square mile of area in the county, and $\bar{A}$ is the average area of all such fields in the county, expressed in square miles. The numerical values of c_2 have been given previously in this report. The derivation of equation (5) may be found in the Appendix.

The observed numbers of fields per mile of route and the corresponding numbers computed by means of equation (5) are shown in table 7. The numbers given refer to counts on only one side of a route.

Table 7.—Observed and Computed Numbers of Fields Per Mile of Route

County	Corn		Wheat		Alfalfa		Total	
	Observed number	Computed number	Observed number	Computed number	Observed number	Computed number	Observed number	Computed number
Dane, Wis.	1.447	1.335	0.028	0.041	0.816	0.908	2.291	2.284
McLeod, Minn.	1.030	1.145	.391	.501	.796	.797	2.217	2.443
Washington, Ind.	1.081	.938	.417	.394	.177	.128	1.675	1.460
Gentry, Mo.	.775	.979	.436	.602	.085.	.135	1.296	1.716
Rock, Wis.	1.528	1.482	.103	.084	.259	.270	1.890	1.836
Wayne, Ind.	1.339	1.568	.755	.880	.448	.557	2.542	3.005
Jefferson, Iowa	1.418	1.292	.256	.237	.049	.060	1.723	1.589
Harlan, Nebraska	.984	.824	1.079	.855	.111	.109	2.174	1.788

The observed numbers of fields for each crop are plotted against the corresponding computed values in figure 9. It is quite apparent that the agreement is satisfactory. This indicates that the differences in the values of c_2, from crop to crop, are due to differences in the probability of encountering certain crops on highways and are not spurious effects caused by such factors as differences in field shape.

Observed and Expected Standard Error of a Relative Frontage

The results presented thus far show good agreement between observation and theory. The hypothesis may be subjected to a more stringent test than any of those previously considered by deducing the expression for the standard error of a relative frontage from the theory and comparing the observed variability with the variability which would be expected if the hypothesis under consideration were completely valid in all respects.

To deduce the formula for the standard error of a relative frontage from theoretical considerations, consider a tract of land containing to the square mile n_1, n_2, —, n_p fields whose individual areas, expressed in square miles, are A_1, A_2, —, A_p. Consider the n_1 fields of area A_1.

Each square mile may be divided into $1/A_1$ equal square spaces, each of area A_1. When one of these spaces is taken at random, the probability that it will contain one of the n_1 fields is $n_1 \div 1/A_1$ or $n_1 A_1$. However, if there is a tendency for the crop to be located on the road or away from the road, the probability that a space will contain a field is $c_2 n_1 A_1$. By taking a random route of length k miles one is, in effect, taking a sample of $\frac{k}{\sqrt{A}}$, such spaces, since this is the number of such spaces that can be placed side by side on a route k miles long. The expected number of fields on k miles of route is the

FIGURE 9

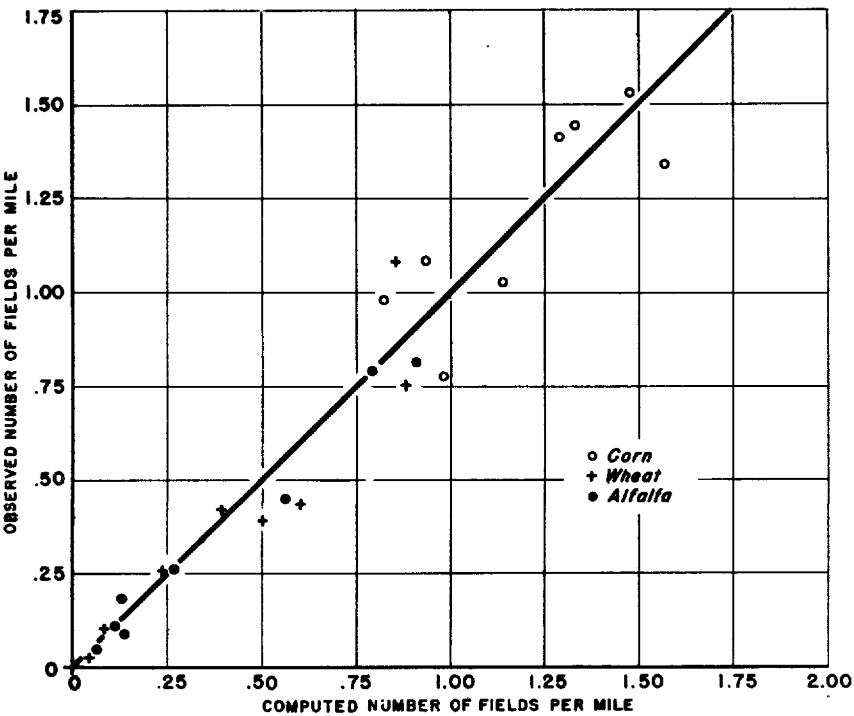

Figure 9.—Observed and computed numbers of fields per mile of route in each county. The position of each dot represents the computed number of fields growing a given crop that was expected on a mile of route in a county as compared with the average number actually found. The straight line through the origin is the expected line of best fit if no bias is present. It was not fitted to the data.

product of $c_2 n_1 A_1$ and $k/\sqrt{A_1}$ or $kc_2 n_1 \sqrt{A_1}$ and the variance of the number of fields on k miles of route is $kc_2 n_1 \sqrt{A_1}(1 - c_2 n_1 A_1)$. Since the number of fields having exactly the area A_1 may be considered small in relation to the total number of fields, the quantity in parentheses is nearly equal to unity and the variance may be taken simply as $kc_2 n_1 \sqrt{A_1}$. The variance of the number of fields on k miles of route, therefore, is equal to the expected number of fields.

The expected frontage of each of these fields is $c_1 \sqrt{A_1}$ but the frontages of individual fields of a given area are subject to a certain amount of variability. Table 8 gives the variance of the 100 fields, previously considered, on the random route for each county investigated and shows that the variance is proportional to the average area of the fields. The data are presented graphically in figure 10.

Table 8.—Variance of Frontage of 100 Individual Fields in Each County

County	Variance of frontages FEET	Average area of fields ACRES	Variance / Average area
Dane, Wis.	117,162	5.127	22,852
McLeod, Minn.	137,626	7.559	18,207
Washington, Ind.	193,756	9.579	20,227
Gentry, Mo.	225,008	11.699	19,233
Rock, Wis.	272,018	12.244	22,216
Wayne, Ind.	147,332	14.172	10,396
Jefferson, Iowa	186,152	16.959	10,977
Harlan, Nebraska	757,659	35.748	21,194
Average			18,162.8

FIGURE 10

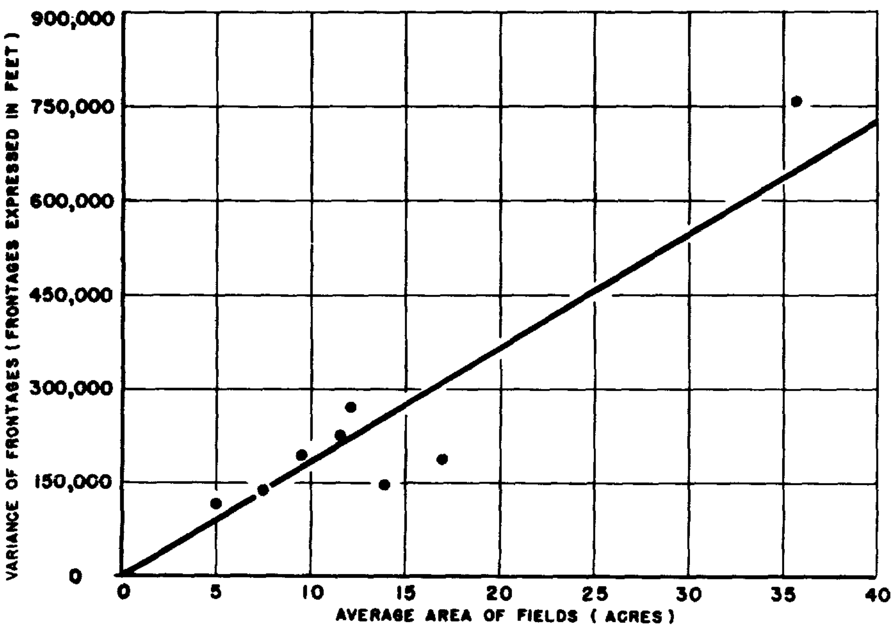

Figure 10.—Relation between variance of frontages and average area of individual fields. The position of each dot represents the variance of the frontages of 100 individual fields on a random route in a county as compared with the average area of those fields. Each dot represents data for a different county. The slope of the straight line through the origin was computed by taking the unweighted average of the ratios of the 8 variances to the corresponding 8 average field areas.

When frontages are expressed in miles and areas in square miles, the average factor of proportionality in table 8 is equal to 0.41696. This constant is denoted by c_3 in the following discussion.

The variance of the total frontage on k miles of route consists of two components. The first is due to the effect of variation in the number of fields and is equal to $(c_1\sqrt{A_1})^2 (k\, c_2\, n_1\sqrt{A_1})$ or $k\, c_1^2\, c_2 n_1 A_1^{3/2}$. The second is due to the variability of the frontages of fields of the same area and is approximately equal to $(kc_2 n_1 \sqrt{A_1})(c_3 A_1)$ or $kc_2c_3n_1A_1^{3/2}$. The variance of the total frontage of fields of area A_1 is equal to the sum of these two components or $k(c_1^2 + c_3)c_2n_1A_1^{3/2}$.

The variance of the total frontage contributed by all the fields is obviously equal to $k\,(c_1^2 + c_3)\,c_2\,S\,(n_i\,A_i^{3/2})$, an expression which can be evaluated by integration. The variance of the relative frontage is obtained by dividing the result by k^2. When these operations are performed, the formula for computing the variance of a relative frontage, based on k miles of route, can be written in the form,

$$s^2 = \frac{1.0838(c_1^2 + c_3)c_2 n\bar{A}^{3/2}}{k} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

The number of fields of a given crop per square mile of area is represented by n and the average area of those fields, expressed in square miles, is represented by $\bar{A}$.

Equation (6) was tested by comparing results given by the formula with the discrepancy between the observed and expected relative frontages for each crop in each county. If the relative area of a crop acreage in a given county is written in the form $n\bar{A}$, the expected relative frontage of the crop on routes is equal to $c_1\, c_2\, n\, \bar{A}$. Values of this quantity can be computed for each crop in each county and compared with the corresponding observed values. The sum of the squares of the differences each divided by the corresponding variance as computed from equation (6), should be distributed as chi square if the theory is valid.

Preliminary work on this problem showed that the use of only three different values of c_2, correcting for crop bias, were not sufficient to reduce the variability of the relative frontages to an amount comparable with the variances computed by equation (6). Therefore eight additional factors correcting for county bias were computed and used in the analysis. The proper value of k to be used in equation (6) is twice the total miles driven in each county, since the relative frontages observed are based on observations taken on both sides of every route.

If the computed acreages in table 3 are multiplied by $\frac{880742.}{804842}$ the grand total of the computed acres in the table will be brought into agreement with the total of the observed acres, though the various crop and county totals will not be equal. Therefore, the adjusted computed acreage for each crop in each county must be multiplied by an additional factor of the form $t_i t_j$ in which the t_i represent the three crop bias factors and the t_j represent the eight county bias factors.

Although it is theoretically possible to compute the values of these constants from the data, such a computation would involve the solution of equations of considerable complexity. Since the t_i and t_j do not differ greatly from unity, a serviceable approximation is obtained by writing $t_i t_j = (1 + x_i)(1 + y_j)$ which is approximately equal to $1 + x_i + y_j$, when the x_i and y_j are small. The numerical values of the x_i and y_j were computed from the data so that when factors of the form $1 + x_i + y_j$ were used as multipliers of the computed acres, as adjusted above, the crop and county totals were equal to the corresponding observed totals. A comparison of the final adjusted computed acreages with the corresponding observed acreages is given in table 9.

Table 9.—Observed Crop Acreages and Computed Acreages Adjusted for Crop and County Bias

County	Corn		Wheat		Alfalfa		Total	
	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres	Observed acres	Computed acres
Dane, Wis.	138,415	143,137	2,799	1,954	48,354	44,475	189,568	189,566
McLeod, Minn.	57,832	55,218	20,837	18,139	15,568	20,879	94,237	94,236
Washington, Ind.	36,609	39,067	17,266	14,658	2,986	3,136	56,861	56,861
Gentry, Mo.	45,047	46,815	29,005	27,107	2,968	3,098	77,020	77,020
Rock, Wis.	107,439	106,304	4,121	5,734	9,120	8,642	120,680	120,680
Wayne, Ind.	63,636	63,538	36,825	36,542	10,740	11,121	111,201	111,201
Jefferson, Iowa	62,149	62,301	9,512	9,852	1,239	747	72,900	72,900
Harlan, Nebraska	71,678	66,421	83,222	89,603	3,375	2,251	158,275	158,275
Total	582,805	582,801	203,587	203,589	94,350	94,349	880,742	880,739

A comparison of the observed and expected relative frontages, after adjusting for crop and county bias, is given in table 10.

Table 10.—Comparison of Observed and Expected Relative Frontages

County	Corn			Wheat			Alfalfa		
	Relative frontage observed	Relative frontage expected	Diff. / S.E.	Relative frontage observed	Relative frontage expected	Diff. / S.E.	Relative frontage observed	Relative expected frontage	Diff. / S.E.
Wis.	0.155653	0.150518	+0.782	0.002170	0.003109	−1.182	0.080801	0.087848	−1.549
d, Minn.	.136528	.142990	− .655	.045749	.052052	−1.124	.082571	.061566	+4.123
ington, Ind.	.127828	.119786	+1.122	.049244	.058006	−1.605	.020332	.019360	+ .344
y, Mo.	.099577	.095816	+ .583	.058627	.062731	− .743	.009620	.009217	+ .228
Wis.	.182434	.184381	− .212	.010039	.007214	+1.839	.023631	.024938	− .449
Ind.	.193214	.193514	− .032	.113170	.114046	− .093	.051619	.049851	+ .336
Iowa	.210707	.210193	+ .045	.034119	.032941	+ .272	.004557	.007559	−1.647
Neb.	.182360	.196794	−1.045	.252314	.234335	+1.090	.011787	.017670	−1.881

The sum of the squares of ratios of the differences to their standard errors should be equal to a value of chi square corresponding to 14 degrees of freedom. For the given data, chi square is equal to 40.36 which is excessively large for 14 degrees of freedom. It should be noted that the discrepancy between observed and expected frontage of alfalfa in McLeod county alone contributes 17 units to this value. All of the other contributions to the value of chi square appear to be of the expected order of magnitude.

It is tempting to explain the one highly aberrant observation in the table as an accidental occurrence caused by nothing more than fluctuations under random sampling. However, it is highly probable that the crop and county bias factors used in the above analysis are not adequate for determining the true bias in the relative frontage for a given crop in a given county. It seems reasonable to suppose that a crop bias need not necessarily be constant from county to county or that a county bias need not necessarily be constant for all crops. The conclusion to be drawn from the above results is that equation (6) provides a valid estimate of the variance of a relative frontage when sources of bias are removed. In actual practice, crop meter routes are chosen in such a way that bias in the results is constant from year to year, and under such conditions equation (6) is likely to yield satisfactory results.

Summary and Conclusions

The results of the study herein reported show that the relative frontage of a crop on highways in a county is proportional to the relative acreage of that crop in the county. The factor of proportionality varies from crop to crop and from county to county because of a tendency for certain crops to be planted near highways and because of differences in the topography and utilization of the land traversed by the routes. The factors of proportionality for various crops and localities do not differ greatly from unity. That fact lends considerable support to the hypothesis that, on the average, areas of individual fields tend to be equal to the squares of their respective frontages and that the chance of encountering a field on a mile of route tends to be equal to the square root of its area, the area being expressed in square miles. Such discrepancies as were observed can be explained by sources of bias, like those noted above, which have been recognized for some time by users of the crop meter. A detailed analysis of the data confirmed this point of view.

In the practical use of the crop meter, the effects of some of these sources of bias can be eliminated by using the year-to-year change in relative frontage on identical routes as a measure of change in relative acreage. This practice is already being followed by most users of the crop meter. The effects of variability in the type of farming area traversed can be eliminated by proper stratification of the area under consideration.

The success of the crop meter depends largely upon the postulate that the region traversed be fairly homogeneous with respect to distribution of crops and topography of the land. Whenever it is possible to distinguish different types of farming regions within a county or State, and the boundaries of such regions can be defined, it seems desirable to obtain relative frontage measurements separately for each type of area and to apply them separately in computing the crop acreages even though identical routes are used from year to year. The same result could be achieved by laying out the routes in such a manner that the distance driven through each type of region would be proportional to the geographic area of the region involved. This would lead to a properly weighted average relative frontage that could be applied to the county or State as a whole.

It also appears to be important to continue the present practice of rigorously defining what constitutes a crop frontage. Such a definition should be based on considerations involving ease of application on the part of the operator engaged in making the measurements rather than on theoretical grounds. Any sources of bias introduced by the definition of a crop frontage should be constant and so long as other known constant sources of bias are present no additional difficulties will be introduced. From the point of view of mathematical theory, the nature of the definition is not important. The important consideration is that the definition be followed consistently and the one easiest to apply should be the one put into practice. An operator in a moving automobile, observing crops and seeing that they are recorded on the crop meter, is not likely to make efficient use of a complicated definition.

Mathematical Appendix

The frequency distribution of fields in a county is assumed to be

$$df_A = \frac{a^{b+1}}{\Gamma(b+1)} e^{-aA} A^b dA$$

If the probability of encountering a field on a mile of route is proportional to the square root of its area, the frequency distribution of the fields on the routes is

$$df_{A_r} = \frac{a^{b+3/2}}{\Gamma(b+3/2)} e^{-aA_r} A_r^{b+1/2} dA_r$$

The constants a and b depend upon the average size of the fields in the county and the coefficient of variation. Applying the method of moments, the values of the constants are found to be

$$a = \frac{1}{\bar{A}v^2}, \qquad b = \frac{1}{v^2} - 1$$

in which v^2 is expressed as a decimal fraction.

The average area of the fields in the county is,

$$\bar{A} = \frac{a^{b+1}}{\Gamma(b+1)} \int_0^\infty e^{-aA} A^{b+1} dA = \frac{b+1}{a}$$

The average area of the fields on the routes is

$$\bar{A}_r = \frac{a^{b+3/2}}{\Gamma(b+3/2)} \int_0^\infty e^{-aA_r} A_r^{\,b+3/2}\, dA_r = \frac{b+3/2}{a}$$

$$\frac{2(\bar{A}_r - \bar{A})}{\bar{A}} = \frac{2}{\bar{A}}\left[\frac{b+3/2}{a} - \frac{b+1}{a}\right] = \frac{1}{a\bar{A}} = v^2$$

If the probability of getting a field of area A in a mile of route is $c_2\sqrt{A}$ and the total number of fields per square mile of area is n, the expected number of fields per mile of route is

$$n_r = \frac{c_2 n a^{b+1}}{\Gamma(b+1)} \int_0^\infty e^{-aA} A^{b+1/2} dA = \frac{c_2 n \Gamma(b+3/2)}{a^{1/2}\Gamma(b+1)}$$

If v is taken as equal to 0.48048, this expression reduces to

$$n_r = 0.97163\, c_2\, n \sqrt{\bar{A}}\ .$$

Under the assumption of a homogeneous distribution of fields, the variance of a relative frontage based on k miles of route was shown to be equal to

$$\frac{1}{k}(c_1^2 + c_3)c_2\ S(n_i A_i^{3/2})$$

in which n_i is the number of fields of area A_i per square mile. If $S(n_i) = n$, one obtains,

$$S(n_i A_i^{3/2}) = \frac{n a^{b+1}}{\Gamma(b+1)} \int_0^\infty e^{-aA} A^{b+3/2} dA = \frac{n\Gamma(b+5/2)}{a^{3/2}\Gamma(b+1)}\ .$$

Taking v as equal to 0.48048, this expression reduces to,

$$S(n_i A_i^{3/2}) = 1.0838\ n\ \bar{A}^{3/2}$$

and the variance of the relative frontage reduces to

$$1.0838(1/k)(c_1^2 + c_3)c_2\, n\, \bar{A}^{3/2}.$$